

# HADRONIZATION IN THE CHROMODIELECTRIC MODEL

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## 1 Introduction

Quantumchromodynamics (QCD) is the widely accepted theory to describe strong interactions between hadrons. This theory shows the well-known behavior of *asymptotic freedom*. Furthermore, lattice calculation show a phase transition from the hadronic world to a system of free moving quarks and gluons, the Quark-Gluon-Plasma (QGP). Heavy-ion experiments at CERN-SPS, at recently started BNL-RHIC and at the yet to come CERN-LHC with energies of  $\sqrt{s} = 20, 200, 5500$  A GeV respectively are designed to study the eventually formed QGP. But still there is lack of a dynamical description of both the transitions from hadrons to quarks and gluons and vice versa, derived from first principles from QCD. In this talk I present a classical, molecular-dynamical model, which contains explicitly the phenomenon of confinement and a dynamical mechanism for the formation of hadrons out of large system of quarks and gluons.<sup>1,2,3,4</sup>

## 2 The chromodielectric model (CDM)

We start with the Lagrange-density originally invented by Friedberg and Lee<sup>5,6</sup> and intensively studied by several followers<sup>7,8,9,10</sup>.

$$\begin{aligned} \mathcal{L} = & \bar{q} (i\gamma_\mu D^\mu - m) q - \frac{1}{4} \kappa(\sigma) F_{\mu\nu}^a F^{\mu\nu a} \\ & + (\partial_\mu \sigma)(\partial^\mu \sigma) - U(\sigma) \quad . \end{aligned} \quad (1)$$

The first term describes quarks, where  $q$  is a Dirac spinor with color-, spinor- and flavor-indices being suppressed and  $m$  is a mass-matrix for the different quark-flavor.  $D^\mu = \partial^\mu + igA^\mu$  is the covariant derivative, describing the minimal coupling to the gauge fields  $A^\mu = \frac{\lambda^a}{2} A^{\mu a}$  with coupling constant  $g$  and  $\lambda^a$ ,  $a = 1 \dots 8$ , being the Gell-Mann matrices.

The second term is the kinetic term for the gauge field in a medium, mediated via a dielectric function  $\kappa(\sigma)$ . The color field tensor is given by  $F_{\mu\nu}^a = \partial_\mu A_\nu^a -$

$\partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$ , where the  $f^{abc}$  are the structure constants of  $SU(3)_c$  and one has  $-\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} = \frac{1}{2}(\vec{E}^a \vec{E}^a - \vec{B}^a \vec{B}^a)$ .  $\vec{E}$  and  $\vec{B}$  are the color-electric and -magnetic fields.

The last two terms introduce a scalar field  $\sigma$  with a quartic scalar potential  $U(\sigma)$ . This scalar field is designed to mimic the long-range behavior of non-perturbative QCD and is therefore purely classical. It acts like a medium as in classical electrodynamics but with a dielectric constant  $\kappa(\sigma) < 1$ <sup>11</sup>. The potential  $U(\sigma)$  is adjusted to have a global minimum at the vacuum expectation value (VEV)  $\sigma = \sigma_v$  and a local minimum at  $\sigma = 0$ . In the absence of color-fields, the scalar field takes on its VEV everywhere.

### 2.1 Confinement mechanism

The mechanism of confinement in the CDM is based on an interplay of the color-fields and the  $\sigma$ -field via the dielectric function  $\kappa(\sigma)$  and the scalar potential which are shown schematically in fig. 1. From the Lagrangian (1) one gets the equations of motion for the color-field.

$$[D_\mu, \kappa(\sigma) F^{\mu\nu}] = j^\nu \quad , \quad (2)$$

where  $j^\nu = g \bar{q} \gamma^\nu \lambda^a q \lambda^a / 2$  is the color-current of the quarks.<sup>a</sup> In an Abelian approximation the equations for the color-fields reduce to the usual Maxwell-equations  $\partial_\mu (\kappa(\sigma) F^{\mu\nu a}) = j^{\nu a}$ . The crucial point of the model is the choice of  $\kappa(\sigma)$ , which is supposed to contain all non-Abelian effects. It is unity in the absence of the  $\sigma$ -field and it vanishes when the scalar field takes on its VEV  $\sigma_v$ . If one considers a color-charge-distribution  $\rho$  with vanishing total color projections (in the Abelian directions 3 and 8), which we call a white cluster, a color-field is produced due to the Gauss-law  $\vec{\nabla} \cdot (\kappa(\sigma) \vec{E}^a) = \rho^a$ . The field is only allowed where  $\kappa(\sigma) > 0$  ( $\sigma < \sigma_v$ ). To suppress the scalar field costs an energy  $U(0) = B$ , and the vacuum exerts a pressure on the color-field. If the transition from  $\kappa = 1$  to  $\kappa = 0$  is a rapid one, then one is left with a well defined spatial region, where the scalar field nearly vanishes and the color field is non-zero. All color-field-lines start and end on charges inside this volume and therefore there are no Van-der-Waals-like interactions to other white clusters except for very short-ranged  $\sigma$ -effects. For that reason, if eventually two white subclusters form inside the cluster, they can separate from each other.

In addition, if the charge-distribution has a non-vanishing total charge, the field-energy deposited in this configuration is divergent, i. e. those configuration cannot be created.

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<sup>a</sup>Note that this current is not conserved, due to the color-carrying gluons.

## 2.2 Model equations

As the confinement mechanism in our model depends solely on the specific choice of the dielectric function  $\kappa(\sigma)$  and the scalar potential  $U(\sigma)$ , we can neglect the spin-dependences in the quark-Lagrangian. Instead we replace it with a Lagrangian for classical, spinless charged particles that are coupled to the classical color-field.

$$\mathcal{L}_p = \sum_k m_k \sqrt{1 - \dot{\vec{x}}_k^2} \rho_N(\vec{x} - \vec{x}_k) - j^{\mu a} A_\mu^a \quad (3)$$

$$j^{\mu a} = g \sum_k q_k^a u_k^\mu \rho_N(\vec{x} - \vec{x}_k) \quad (4)$$

with the color-charge  $q_k^a$  and the 4-velocity  $u_k^\mu$ . In our numerical realization we deal with extended particles with a fixed Gaussian distribution

$$\rho_N(\vec{x} - \vec{x}_k) = (2\pi r_0^2)^{-3/2} e^{-(\vec{x} - \vec{x}_k)^2 / (2r_0^2)} \quad , \quad (5)$$

where  $\sqrt{\langle \vec{x}^2 \rangle} = \sqrt{3}r_0 = 0.7 \text{ fm}$  is chosen to fit the radius of the nucleon.

The color-current (4) is consistent with the originally derived equations of motion (2) only in an Abelian sub-group of  $\text{SU}(3)_c$ . This is related to the fact, that the gluons in QCD carry color as well, whereas the color-current carried by the gluons vanishes in the Abelian approximation<sup>2,12</sup>.

To further simplify the numerical realization we neglect the magnetic fields  $\vec{B}^a$ . This is exact for static problems and for string like yoyo-excitations<sup>13</sup>. The two decoupled sets of Maxwell-equations reduce basically to the Gauss-law for each field. To summarize we now have the following equations of motion for the particles, the (electric) color-field and the  $\sigma$ -field

$$\dot{\vec{x}}_k = \frac{\vec{p}_k}{E_k} \quad (6)$$

$$\dot{\vec{p}}_k = -q_k^a \int d^3x \left( \vec{\nabla} \phi^a(\vec{x}) \right) \rho_N(\vec{x} - \vec{x}_k) \quad (7)$$

$$\vec{\nabla} \cdot \left( \kappa(\sigma) \vec{\nabla} \phi^a(\vec{x}) \right) = -\rho^a(\vec{x}) \quad (8)$$

$$\frac{\partial^2 \sigma}{\partial t^2} + U'(\sigma) = \nabla^2 \sigma + \frac{1}{2} \kappa'(\sigma) \left( \vec{\nabla} \phi^a(\vec{x}) \right) \cdot \left( \vec{\nabla} \phi^a(\vec{x}) \right) \quad , \quad (9)$$

where the prime denotes differentiation with respect to  $\sigma$  and  $\phi^a(\vec{x})$  is the electric potential which satisfies  $\vec{E}^a = -\vec{\nabla} \phi^a(\vec{x})$ ,  $a \in \{3, 8\}$ . We choose for the scalar potential  $U(\sigma) = B + a\sigma^2 + b\sigma^3 + c\sigma^4$  with  $B = (150 \text{ MeV})^4$ ,  $a = (489.9 \text{ MeV})^2$ ,  $b = -15901 \text{ MeV}$ ,  $c = 163.1$  and for the VEV  $\sigma_v = 61.1 \text{ MeV}$ .

The dielectric function is  $\kappa(\sigma) = \left( \exp \left( \alpha \left( \frac{\sigma}{\sigma_v} - \beta \right) \right) + 1 \right)^{-1}$ , where  $\alpha = 7$  and  $\beta = 0.4$ . The coupling constant is chosen to reproduce the string-tension in an  $q\bar{q}$ -configuration and takes on the value  $\alpha_s = g/(4\pi) = 2$ .

### 2.3 Classification of particles

In QCD the quarks and gluons are represented as triplet- and octet-states of  $SU(3)_c$  respectively. In our classical simulation we assign classical charges  $q^a$  to the quarks. These charges are the diagonal entries of the  $\lambda^a, a \in 3, 8$ . Due to our approximation we neglect the non-Abelian part of the color-fields. Instead we treat these 6 gluon-fields as particles in the same formalism as the quarks except that these particle-gluons carry both a color and an anti-color. The corresponding charges for quarks, anti-quarks and gluons are depicted in fig. 2. E.g. the particle  $r$  in that scheme is a quark of color *red* and color-charges 1 and  $1/\sqrt{3}$  with respect to the 3- and the 8-field respectively.

As we have mentioned in section 2.1, the dynamics of our model forces the charged particles into white clusters and the separation into white subclusters is allowed. We regard as hadrons only white clusters which cannot be divided into smaller ones. It turns out, that there is only a finite set of those *irreducible white Clusters* (IWC). The IWCs consist either of a quark and an anti-quark or of three quarks (anti-quarks), both with some gluon admixture, or they consist only of gluons and can therefore easily be interpreted as mesons, baryons and glue-balls respectively.

For the particle masses we use constituent quark masses to fit the low-lying hadronic spectrum. As our model does not depend on isospin, we treat the  $u$ - and the  $d$ -quark as degenerate particles with the same mass. Only the light pion, which is assumed to be a Goldstone-boson of chiral symmetry breaking, does not fit in our constituent-mass-scheme and thus we do not incorporate pions in our model. The quark masses are fixed to be  $m_{u,d} = 400\text{MeV}$ ,  $m_s = 550\text{MeV}$  and  $m_c = 1500\text{MeV}$ . Thus the masses of the lowest hadronic states are simply the sum of the quark masses of the IWC. For the gluons we take a mass  $m_g = 700\text{MeV}$  to reproduce the lightest glueball-mass of  $1400\text{MeV}$ .

## 3 Hadronization

To simulate the hadronization out of a QGP, we start with an ensemble of quarks and massive gluons, distributed homogeneously in a sphere of radius  $R = 4\text{fm}$  in real space and according to a Boltzmann-distribution with initial temperature  $T_0 = 160\text{MeV}$  in momentum-space. The relative number of different particles is given through the distribution  $N_i \propto$

$d_i \exp(-m_i/T_0) \left(\frac{m_i T_0}{2\pi}\right)^{3/2}$ , where  $d_i$  is a (spin, isospin and color) degener-  
ation factor. The colors are chosen randomly with the constraint of overall  
color-neutrality. After solving the Gauss-law for the color-fields in the first  
time step the system is driven by the equations of motion (6). Due to the  
initial momenta the particles tend to leave the system but are bound due to  
the formation of color-strings. In this way the particles are reorganized to  
form in a first step white clusters and finally only IWCs. This scenario is  
shown in fig. 3. The hadrons have invariant masses  $M_{\text{iwc}}^2 = E^2 + \vec{P}^2$  with  
particle energies  $E_i$  and momenta  $\vec{p}_i$

$$E = \sum_i E_i + \int d^3x \left( \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} (\vec{\nabla} \sigma)^2 + U(\sigma) + \frac{1}{2} \kappa(\sigma) \vec{E}^a \vec{E}^a \right) \quad (10)$$

$$\vec{P} = \sum_i \vec{p}_i - \int d^3x \dot{\sigma} \vec{\nabla} \sigma \quad (11)$$

The resulting mass distribution is shown in fig. 4. The curves are fits to  
a Hagedorn distribution  $dn/dm \propto m^{a+3/2} \exp(-m(1/T - 1/T_h))$  and to an  
inverse power-law-distribution  $dn/dm \propto m^{-\tau}$ . If we assume  $T_h = 160\text{MeV}$   
and  $a = -3$  or  $a = -3/2$  in the Hagedorn case, we get a hadronic temperature  
 $T = 146\text{MeV}$  and  $T = 125\text{MeV}$  respectively.

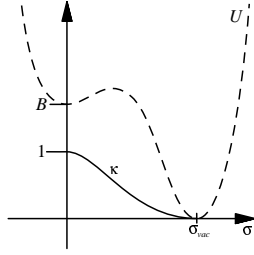


Figure 1. The scalar poten-  
tial  $U(\sigma)$  and the dielectric  
function  $\kappa(\sigma)$ .

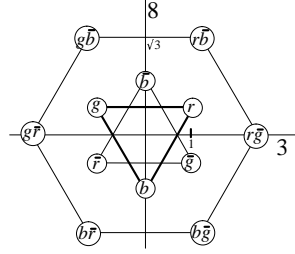


Figure 2. The color scheme in the Abelian approximation.  
On the 3(8)-axis is given the particle charge with respect  
to the 3(8)-field. The particle on the two triangles (r, g,  
b) and ( $\bar{r}$ ,  $\bar{g}$ ,  $\bar{b}$ ) represents the quarks and anti-quarks and  
those on the hexagon the 6 massive gluons.

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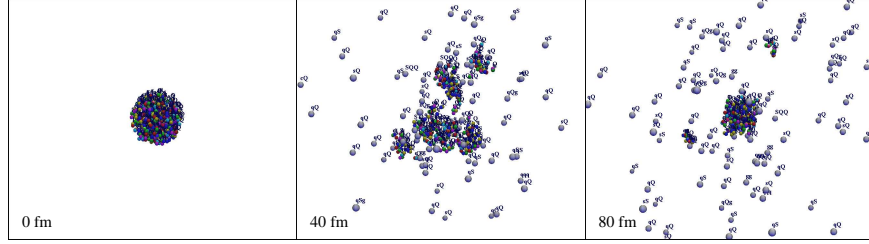


Figure 3. Different timesteps of the hadronization process. The simulation starts with a droplet of quarks and gluons and ends after about 200 fm left with IWCs only. The particle labels describe the flavor content and grey shaded particles are already detected IWCs.

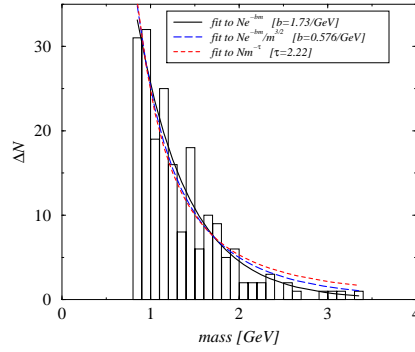


Figure 4. The mass distribution of the resulting hadrons. The curves are fits to a Hagedorn distribution with  $a = -3/2$  and  $a = -3$  and  $b = (1/T - 1/T_h)$  and to an inverse power-law-distribution.

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